Step-by-Step Evaluation of the Holomorphic Unified Field Theory Against the Speed of Light in Vacuum

October 5, 2025

The Holomorphic Unified Field Theory (HUFT), as detailed in the arXiv paper (2506.19161), unifies gravity and the Standard Model (including electromagnetism) via a single holomorphic action on a four-complex-dimensional manifold. The real 4D slice reproduces general relativity (GR) from the symmetric part of the Hermitian metric $g_{(\mu\nu)}$ and Maxwell's equations from the antisymmetric part $g_{[\mu\nu]}$, identified with the electromagnetic field strength $F_{\mu\nu}$. The speed of light in vacuum c is the invariant speed for null geodesics in GR and the propagation speed for electromagnetic waves in Maxwell's theory. The experimental value is $c = 299,792,458$ m/s (exact by definition in SI units since 2019).

I will test HUFT by deriving the electromagnetic wave speed from its equations and comparing it to the known c. If the deviation exceeds 1%, I will amend the theory (e.g., by rescaling couplings or introducing corrections) and retest. All derivations assume natural units where $c = 1$ initially for simplicity, but I'll restore c for comparison.

1 Step 1: Extract the Metric and Field Strengths from the Holomorphic Framework

The theory starts with a holomorphic Hermitian metric $g_{\mu\nu}(z)$ on $\mathbb{C}^4$ coordinates $z^\mu = x^\mu + iy^\mu$. On the real slice ($y^\mu = 0$):

$$g_{\mu\nu}(x) = g_{(\mu\nu)}(x) + i\, g_{[\mu\nu]}(x),$$

where $g_{(\mu\nu)}$ is the Lorentz metric (signature $-,+,+,+$) governing gravity, and $g_{[\mu\nu]} \propto F_{\mu\nu}$ for electromagnetism. The normalization is set such that $g_{[\mu\nu]} = \frac{e}{2}F_{\mu\nu}$ (with e the elementary charge) to match Standard Model couplings, but for wave propagation, the scaling affects strength, not speed.

- **Transformation**: The holomorphic compatibility condition $\nabla^{(\Gamma)}_\lambda g_{\mu\nu} = 0$ (where Γ is the Christoffel connection) splits into real and imaginary parts. The imaginary antisymmetric part gives the Bianchi identity for $F_{\mu\nu}$:

$$\partial_{[\alpha} g_{[\beta\gamma]]} = 0 \implies \partial_{[\mu} F_{\nu\rho]} = 0.$$

  This is invariant under coordinate transformations and preserves the Lorentz structure.

No deviation yet; this is standard.

2 Step 2: Derive Maxwell's Equations in the Real Slice

Variation of the effective 4D action $S = \int d^4x \sqrt{-g}\left(R(g_{(\mu\nu)}) - \frac{1}{4} g^{[\mu\nu]} g_{[\mu\nu]} + \cdots\right)$ (truncated to EM terms) yields the sourced Maxwell equations:

$$\nabla^\rho g_{[\rho\nu]} = J_\nu \implies \nabla^\rho F_{\rho\nu} = \frac{2}{e} J_\nu,$$

where J^ν is the four-current. In vacuum ($J^\nu = 0$):

$$\nabla^\rho F_{\rho\nu} = 0, \quad \partial_{[\mu} F_{\nu\rho]} = 0.$$

- **Transformation to Wave Equation**: In flat spacetime (Minkowski limit, $g_{(\mu\nu)} = \eta_{\mu\nu}$), introduce the vector potential A_μ such that $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Choose Lorentz gauge $\partial^\mu A_\mu = 0$. The vacuum equation becomes:
$$\partial^\rho(\partial_\rho A_\nu - \partial_\nu A_\rho) = 0 \implies \Box A_\nu = 0,$$
where $\Box = \partial^\mu \partial_\mu = -\frac{\partial^2}{\partial t^2} + \nabla^2$ (in units with $c = 1$).

Restoring c: The d'Alembertian is $\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2$, so plane-wave solutions $A_\nu \sim e^{i(k\cdot x - \omega t)}$ satisfy $\omega = c|k|$, giving propagation speed c.

- **Comparison to Known** c: The theory reproduces standard Maxwell's equations exactly, so the derived speed is $c_{\text{theory}} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 299,792,458$ m/s (matching the vacuum permittivity and permeability implicit in the action's normalization). Deviation: 0% (exact match).

3 Step 3: Check for Modifications in Curved Spacetime or Unification Effects

In curved backgrounds (e.g., near masses), light follows null geodesics of $g_{(\mu\nu)}$:

$$ds^2 = g_{(\mu\nu)} dx^\mu dx^\nu = 0 \implies \frac{dx^0}{d\lambda} = c\frac{|d\mathbf{x}|}{d\lambda},$$

where c is the local invariant speed. The holomorphic unification introduces no extra terms that alter the null cone structure, as the imaginary part only sources gauge fields without back-reacting on the speed (confirmed by the paper's claim of exact GR + SM recovery at low energies).

- **Transformation**: If there were a coupling deviation (e.g., from higher-order holomorphic corrections), it might appear as $\Box A_\nu + \kappa R A_\nu = 0$ (where R is curvature), but the paper explicitly states no such terms in the real-slice reduction. Thus, no speed variation.

Deviation remains 0%.

4 Step 4: Quantitative Test Against Experimental Constraints

Experimental tests of c constancy (e.g., Michelson-Morley experiment, precision to ¡$10^{-17}; orastrophysicalobservationslikega$ $raybursts, deviation < 10^{-15})alignwithHUFT'spredictionofinvariantc$. The theory's GUT-scale unification ($10^{16} GeV)introducesnolow - energymodificationstoc$, as running couplings affect strengths (e.g., α), not the speed.

- **Final Deviation Calculation**: $|c_{\text{theory}} - c_{\text{ exp}}|/c_{\text{ exp}} = 0 < 1\%$.

The theory passes without amendment. No new theory is needed, as it fits exactly. If future experiments show variations (e.g., in strong gravity), amendments could involve adding holomorphic scalars, but current data confirms it.