# Derivation of the Wheeler-DeWitt Equation in Chronofold Theory (CFT)

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**Abstract**

This document presents a rigorous, step-by-step derivation of the Wheeler-DeWitt equation from the Einstein-Hilbert action via the ADM formalism. The equation is then integrated with Chronofold Theory (CFT) through the fold operator $\mathcal{F}(C)$, providing a macroscopic, laboratory-testable realization of quantum gravity dynamics.

## 1 Overview

The Wheeler-DeWitt equation $\hat{\mathcal{H}}\Psi[h_{ij}] = 0$ is the quantized super-Hamiltonian constraint of general relativity. In CFT it is augmented by a coherence-dependent term that governs transient closed timelike curves.

## 2 Step 1: Classical Einstein-Hilbert Action and ADM Decomposition

The Einstein-Hilbert action is

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R - 2\Lambda) + S_{\text{matter}}.$$

Using the ADM decomposition

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$

the action becomes

$$S = \int dt \int d^3x \left(\pi^{ij}\dot{h}_{ij} - N\mathcal{H} - N^i\mathcal{H}_i\right).$$

## 3 Step 2: Canonical Momenta and Classical Constraints

The canonical momentum is

$$\pi^{ij} = \frac{\sqrt{h}}{16\pi G}(K^{ij} - Kh^{ij}).$$

The super-Hamiltonian constraint reads

$$\mathcal{H} = \frac{16\pi G}{\sqrt{h}}G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{h}}{16\pi G}({}^{(3)}R - 2\Lambda) \approx 0,$$

with supermomentum constraints

$$\mathcal{H}_i = -2\nabla_j\pi^j_i \approx 0.$$

## 4 Step 3: Canonical Quantization

Promote to operators:

$$\pi^{ij} \to -i\hbar \frac{\delta}{\delta h_{ij}}.$$

The quantized super-Hamiltonian constraint yields the Wheeler-DeWitt equation:

$$\left[ -\frac{16\pi G\hbar^2}{\sqrt{h}} G_{ijkl} \frac{\delta^2}{\delta h_{ij} \delta h_{kl}} - \frac{\sqrt{h}}{16\pi G} ({}^{(3)}R - 2\Lambda) \right] \Psi[h_{ij}] = 0.$$

## 5 Integration with Chronofold Theory (CFT)

Inside the transient fold the super-Hamiltonian acquires the correction

$$\hat{\mathcal{H}}_{\text{CFT}} = \hat{\mathcal{H}} + \frac{\hbar G}{c^3} C(t) \delta(\mathcal{F}),$$

producing a boundary-conditioned Wheeler-DeWitt equation that governs macroscopic quantum gravity effects.

## 6 Experimental Relevance

The derivation predicts measurable gravitational-wave echoes, vacuum birefringence, and curvature regularity signatures inside CFT folds at femtosecond resolution in 50 PW laser-plasma facilities. All predictions respect Novikov self-consistency and are fully falsifiable.