A De Aquino-Compatible Repair of the Weak Gravitational Coupling Proposal

Corrected equations, three adversarial falsification attempts, and a conditional experimental framework

September 2026

Abstract

The April 2026 EquationFarm/Grok paper on weak gravitational coupling contains a crucial transcription or derivational error: its displayed electromagnetic reduction for the gravitational-to-inertial mass ratio is a bounded rational function, yet the paper subsequently requires arbitrarily large magnitude of that ratio. This report repairs the presentation by returning to the correlation actually stated in Fran De Aquino's published manuscripts, namely a nonlinear dependence on normalized momentum transfer. The corrected relation can produce zero gravitational mass, negative gravitational mass, and arbitrarily large *negative* values of the ratio, while it cannot produce values greater than +1 for real momentum transfer. This distinction is sufficient to preserve the paper's proposed strong-coupling algebra for a pair of equally negative effective gravitational masses, because the product of two negative correlation factors is positive and can greatly exceed unity in magnitude. Three attempts are then made to discredit the repaired framework. The first tests algebraic self-consistency and is passed after removing the incorrect Grok reduction. The second tests energy-momentum accounting and requires an explicit distinction between the kinematic correlation postulate and the conventional electromagnetic momentum budget; it passes only as a conditional phenomenological theory, not as an experimentally established law. The third confronts equivalence-principle and inverse-square-law constraints; it passes only if the anomalous correlation is state-dependent and absent in ordinary undriven matter, a condition that must be tested rather than assumed. The resulting framework therefore complements De Aquino's work faithfully while refusing to claim empirical verification that does not yet exist.

1 Purpose and scientific status

The requested goal is to preserve as much of Fran De Aquino's conceptual structure as possible while correcting the parts of the Grok-authored paper that are mathematically inconsistent. "Verification" is used here in two different senses and they must not be conflated. *Internal verification* means that the equations are algebraically consistent and their consequences follow correctly. *Experimental verification* means independent reproducible measurements have established the effect. This paper achieves the former for the repaired model; it does not claim the latter.

The original paper correctly identifies the small proton-scale gravitational coupling,

$$\alpha_G = \frac{Gm_p^2}{\hbar c} \approx 5.9 \times 10^{-39}, \tag{1}$$

but its printed absolute proton-proton force values at $r = 1\,\mathrm{fm}$ are wrong. The correct values are

$$F_G = \frac{Gm_p^2}{r^2} \approx 1.87 \times 10^{-34}\,\mathrm{N}, \tag{2}$$

$$F_C = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} \approx 2.31 \times 10^2\,\mathrm{N}, \tag{3}$$

$$\frac{F_G}{F_C} \approx 8.09 \times 10^{-37}. \tag{4}$$

The ratio survives because the same separation scaling appears in both forces, but the absolute values should be corrected in any revised publication.

2 The central correction: restore De Aquino's actual correlation

A correlation appearing in De Aquino's papers can be written

$$\boxed{\chi_{\mathrm{DA}} \equiv \frac{m_g}{m_{i0}} = 1 - 2\left[\sqrt{1+\left(\frac{\Delta p}{m_{i0}c}\right)^2} - 1\right]} \tag{5}$$

or, defining the dimensionless normalized momentum transfer

$$q \equiv \frac{\Delta p}{m_{i0}c}, \tag{6}$$

as

$$\boxed{\chi_{\mathrm{DA}}(q) = 3 - 2\sqrt{1+q^2}.} \tag{7}$$

This is not the same function as the Grok paper's displayed rational approximation

$$\chi_{\mathrm{Grok}}(x) = \frac{1-x}{1+x}. \tag{8}$$

For $x \geq 0$ the Grok expression satisfies $-1 < \chi_{\mathrm{Grok}} \leq 1$ and therefore cannot support the paper's later claim $|\chi| \gg 1$. The incompatibility is algebraic, not interpretive.

Equation (7), by contrast, has the properties

$$\chi_{\mathrm{DA}}(0) = 1, \tag{9}$$

$$\frac{d\chi_{\mathrm{DA}}}{dq} = -\frac{2q}{\sqrt{1+q^2}} \leq 0 \quad (q \geq 0), \tag{10}$$

$$\chi_{\mathrm{DA}}(q) = 0 \quad \Longleftrightarrow \quad q = \frac{\sqrt{5}}{2} \approx 1.118, \tag{11}$$

$$\chi_{\mathrm{DA}}(q) = -1 \quad \Longleftrightarrow \quad q = \sqrt{3} \approx 1.732, \tag{12}$$

$$\chi_{\mathrm{DA}}(q) \sim 3 - 2|q| \quad (|q| \gg 1). \tag{13}$$

Thus the De Aquino relation permits arbitrarily large *negative* χ_{DA}, but never $\chi_{\mathrm{DA}} > 1$ for real q. This is the first major repair to the Grok presentation.

2.1 Inversion formula

The momentum-transfer parameter required for a selected correlation factor follows directly from Eq. (7):

$$\boxed{q(\chi_{\mathrm{DA}}) = \sqrt{\left(\frac{3-\chi_{\mathrm{DA}}}{2}\right)^2 - 1}}, \qquad \chi_{\mathrm{DA}} \leq 1. \tag{14}$$

Examples are

Target χ_{DA}	Required $q = \Delta p/(m_{i0}c)$
0	1.1180
−1	1.7321
−10	6.4226
−100	51.4903
−1000	501.4990

The numbers make clear that very large negative χ_{DA} requires very large normalized momentum transfer in the literal interpretation of Eq. (5).

3 Repair of the strong-coupling argument

The Grok paper attempted to define an effective interaction

$$F'_G = G\frac{(\chi_1 m_{i1})(\chi_2 m_{i2})}{r^2} = (G\chi_1\chi_2)\,\frac{m_{i1}m_{i2}}{r^2}, \tag{15}$$

thereby introducing

$$\boxed{G_{\text{eff}} = G\chi_1\chi_2.} \tag{16}$$

As algebra, Eq. (16) does not require either χ_1 or χ_2 to be positive. Therefore the corrected De Aquino correlation still permits large *positive* pairwise coupling if both engineered bodies have negative correlation factors:

$$\chi_1 = \chi_2 = -K \quad\Longrightarrow\quad \frac{G_{\text{eff}}}{G} = K^2. \tag{17}$$

For example,

$$\chi_1 = \chi_2 = -10^3 \quad\Longrightarrow\quad G_{\text{eff}} = 10^6 G. \tag{18}$$

This salvages the central strong-coupling algebra without making the false statement that De Aquino's correlation produces $\chi = +10^3$. It does not establish that such a state can be physically created; it only shows that the mathematical route to enhanced pairwise coupling is internally compatible with Eq. (5).

If one body remains ordinary, $\chi_2 = 1$, while the other has $\chi_1 < 0$, then Eq. (16) predicts a sign reversal of the Newtonian interaction. If both have the same negative sign, the product is positive and the interaction is attractive in this phenomenological Newtonian extension. A complete relativistic theory would additionally have to specify how negative gravitational source terms enter the stress-energy tensor and whether stability conditions are respected.

4 Repair of the electromagnetic-driving step

The most damaging part of the Grok paper is not De Aquino's Eq. (5); it is the unsupported replacement of momentum transfer by a simple steady power-density ratio that leads to a different functional form. The repair is to keep the primary quantity measurable:

$$q = \frac{\Delta p_{\text{eff}}}{m_{i0}c}. \tag{19}$$

For an ordinary body receiving a known external impulse J, conventional mechanics gives $\Delta p = J$. For electromagnetic radiation incident on an absorber or reflector, the conventional momentum impulse is constrained by the energy delivered. For absorbed radiation in vacuum,

$$\Delta p_{\text{rad}} \simeq \frac{E_{\text{abs}}}{c}, \tag{20}$$

and for ideal specular reflection it is approximately $2E/c$. Hence a conventional photon-momentum estimate gives

$$q_{\text{conv}} \simeq \frac{E_{\text{abs}}}{m_{i0}c^2}. \tag{21}$$

Equation (21) is extremely small unless the deposited electromagnetic energy approaches the rest energy of the participating inertial mass. Therefore modest ELF power cannot generate $q \sim 1$ through ordinary radiation momentum alone.

To preserve De Aquino's proposal as a testable hypothesis without hiding this scale problem, introduce a dimensionless anomalous transfer factor κ:

$$\boxed{\Delta p_{\text{eff}} = \kappa\, \Delta p_{\text{conv}}, \qquad q = \kappa \frac{\Delta p_{\text{conv}}}{m_{i0} c}.} \tag{22}$$

The orthodox limit is $\kappa = 1$. A De Aquino-type electromagnetic anomaly requires $\kappa \gg 1$ under particular material, frequency, pressure, plasma, or absorption conditions. Crucially, κ is not assigned a favorable value by theory here; it is a measurable response coefficient. This amendment converts an inconsistent power-density shortcut into a falsifiable constitutive hypothesis.

5 A conservative constitutive extension

To model state dependence while keeping ordinary matter compatible with precision tests, write

$$\kappa = 1 + \kappa_{\text{anom}}\, S(\omega, P, T, \sigma, \varepsilon_r, \mu_r, \mathcal{I}), \tag{23}$$

where S is a bounded activation function of drive frequency ω, pressure P, temperature T, conductivity σ, relative permittivity ε_r, relative permeability μ_r, and ionization/plasma state $\mathcal{I}$. Require

$$S \to 0 \tag{24}$$

in ordinary undriven bulk matter. Then

$$\chi_{\text{DA}} = 3 - 2\sqrt{1 + \left[(1 + \kappa_{\text{anom}} S)\, \frac{\Delta p_{\text{conv}}}{m_{i0} c}\right]^2}. \tag{25}$$

This is the minimum De Aquino-compatible repair that simultaneously (i) retains the published mass-correlation equation, (ii) avoids the Grok paper's bounded rational inconsistency, and (iii) exposes the required anomalous enhancement as a parameter that experiments must determine.

6 Falsification attempt 1: algebraic self-consistency

Attack. Can the theory generate the regimes it claims using one and the same correlation law?

Failure of the Grok version. No. Its displayed $\chi = (1 - x)/(1 + x)$ cannot yield $|\chi| > 1$ for $x \geq 0$.

Amendment. Replace that expression with the actual De Aquino momentum-correlation relation, Eq. (5), and propagate its allowed domain consistently. Strong positive pairwise coupling is obtained from two negative factors rather than from unsupported positive $\chi > 1$.

Retest. For any $K > 1$, choose $\chi_1 = \chi_2 = -K$. Eq. (14) produces a real q and Eq. (16) gives $G_{\text{eff}}/G = K^2$. The algebra closes.

Verdict: PASS for internal mathematics after repair.

7 Falsification attempt 2: energy and momentum accounting

Attack. Does the theory obtain a large Δp from a small electromagnetic input without specifying where the momentum and energy originate?

Under conventional electrodynamics, Eq. (21) shows that it does not. A kilogram of matter has rest energy about 9×10^{16} J, so ordinary radiation momentum from laboratory-scale energy input gives negligible q.

Even a milligram has rest energy about 9×10^{10} J. Thus the original "modest power" claim fails if Δp is interpreted as ordinary net mechanical impulse.

Amendment. The repaired model explicitly separates conventional momentum delivery from a hypothesized anomalous internal gravitational-response variable through κ. Energy conservation then becomes an experimental constraint: a non-unit κ must either represent a response variable that does not correspond to net center-of-mass mechanical momentum, or be accompanied by a measurable exchange with another field/reservoir. The model is rejected if a closed energy-momentum budget cannot be constructed from measurements.

A minimal bookkeeping equation is

$$\Delta E_{\text{source}} = \Delta E_{\text{EM}} + \Delta E_{\text{mech}} + \Delta E_{\text{grav}} + \Delta E_X, \tag{26}$$

where E_X denotes any genuinely new sector required by the anomaly. The theory must recover $\Delta E_X = 0$ in the orthodox limit. No net-energy motor claim is accepted unless every term is measured and the source of output work is identified.

Verdict: CONDITIONAL PASS as a phenomenological hypothesis; NOT experimentally verified.

8 Falsification attempt 3: equivalence-principle and null-test constraints

Attack. If m_g/m_i can differ strongly from unity, why have precision free-fall experiments not observed large deviations?

The MICROSCOPE mission reported no violation of universality of free fall at roughly the 10^{-15} level for its test bodies. Modern short-range experiments also strongly constrain additional unscreened forces. Therefore a model in which $\chi_{\text{DA}} \neq 1$ generically for ordinary matter is untenable.

Amendment. In Eq. (25), the activation function S must vanish to high precision outside a narrowly specified driven state. This is not a loophole to be invoked after the fact. The state variables and activation threshold must be declared before testing, and the same apparatus must demonstrate $S \approx 0$ in control states and any claimed $S \neq 0$ in driven states.

The decisive experimental observable is a differential acceleration or weight ratio

$$\eta_{\text{drive}} \equiv \frac{2|a_{\text{on}} - a_{\text{off}}|}{|a_{\text{on}} + a_{\text{off}}|}. \tag{27}$$

A legitimate anomaly must track the predeclared drive-state variables while failing to track electrostatic, magnetic, acoustic, thermal, buoyant, vibrational, and radiometric witness channels.

Verdict: CONDITIONAL PASS only for a strongly state-dependent theory.

9 What the repaired theory now says

The revised model can be summarized in five equations:

$$\chi_{\text{DA}} = 3 - 2\sqrt{1+q^2}, \tag{28}$$

$$q = \frac{\Delta p_{\text{eff}}}{m_{i0}c}, \tag{29}$$

$$\Delta p_{\text{eff}} = \kappa \Delta p_{\text{conv}}, \tag{30}$$

$$\kappa = 1 + \kappa_{\text{anom}} S(\text{state variables}), \tag{31}$$

$$G_{\text{eff}} = G\chi_1\chi_2. \tag{32}$$

The first and fifth equations preserve the central De Aquino/Grok gravitational-mass architecture. The middle three equations repair the unsupported electromagnetic step by making the anomalous response explicit and measurable.

10 Predictions that distinguish the repaired model from ordinary physics

If the model is physically correct, rather than merely algebraically consistent, it predicts all of the following.

1. In a specified activated state, repeated measurements infer a nonzero $\kappa_{\text{anom}}S$ from gravitational observables.
2. The inferred χ_{DA} follows Eq. (7), including the zero crossing and negative branch, instead of an arbitrary voltage-to-weight curve.
3. Switching to a control state with $S \to 0$ restores $\chi_{\text{DA}} \to 1$ within experimental error.
4. Two independently activated source/test masses show an interaction proportional to $\chi_1\chi_2$ if Eq. (16) is correct.
5. Sign combinations obey the product rule: one negative and one positive correlation factor reverse the sign; two negative factors restore attraction while amplifying its magnitude.
6. No net useful work appears without a corresponding measured energy transfer from the drive, gravitational potential, or an independently characterized new sector.

Any reproducible failure of these linked predictions falsifies the repaired framework.

11 Experimental foundation in steps

11.1 Step 1: establish a conventional force baseline

Use a torsion balance, superconducting gravimeter, atom interferometer, or force-compensated precision balance. Before energizing the test cell, establish the full transfer function from known calibration forces to output.

11.2 Step 2: eliminate ordinary electromagnetic force paths

Use conductive shielding, controlled grounding, magnetic characterization, field probes, and polarity reversals. Record electric field, magnetic field, electrode leakage, and charging currents synchronously.

11.3 Step 3: eliminate gas and thermal forces

If low-pressure gas or plasma is used, measure pressure, gas temperature, electrode temperature, and residual gas flow. Repeat with identical thermal loading but electromagnetic parameters outside the proposed activation window.

11.4 Step 4: blind the drive state

Randomize on/off and active/control waveforms. The analysis pipeline should not know the condition labels until force estimates and exclusion cuts are frozen.

11.5 Step 5: fit the correlation law, not merely a weight change

For each state infer χ from the measured gravitational observable and test whether a single q-dependent curve of the form in Eq. (7) describes all data. An arbitrary monotonic change is insufficient.

11.6 Step 6: test sign and pair-product predictions

If a negative branch is observed, construct a second independently controlled cell. Test the four sign combinations $(+,+),(+,-),(-,+),(-,-)$. The pair-force result should follow $\chi_1\chi_2$.

11.7 Step 7: close the energy budget

Measure electromagnetic input, thermal dissipation, mechanical work, and any change in gravitational potential. A motor or propulsion claim must not be advanced before this closure test.

12 Why this complements rather than rejects De Aquino

The corrected paper preserves De Aquino's defining proposal: gravitational and inertial mass are related through a nonlinear correlation factor dependent on a momentum-transfer variable. It also preserves the possibility of null and negative gravitational mass within that formalism. The revision rejects only the later Grok paper's incompatible rational reduction and its unsupported numerical feasibility claims.

The important conceptual repair is that strong effective attraction does not require $\chi > +1$. De Aquino's own negative branch is sufficient: two bodies with $\chi = -K$ produce the product $\chi_1\chi_2 = K^2$ in the phenomenological Newtonian interaction law. This is a cleaner and more faithful complement to De Aquino's framework than asserting parameter values that the displayed equations cannot generate.

13 Limits of the result

No mathematical repair can turn an unreplicated physical claim into an experimentally verified fact. The correlation in Eq. (5) is outside mainstream experimentally established gravitational physics. Precision tests strongly support the equivalence of inertial and gravitational response in ordinary matter. Accordingly, the theory presented here should be read as a De Aquino-compatible *conditional model*: it is internally consistent enough to test, and it states exactly what observations would falsify it.

The strongest result achieved is therefore narrower but scientifically stronger than the original Grok paper: the equations are now mutually compatible, the path to large effective pair coupling is mathematically valid within the assumed correlation, the electromagnetic-driving problem is exposed rather than hidden, and the experimental burden is explicit.

14 Conclusions

The Grok-authored weak-coupling paper fails as written because the equation it uses for electromagnetic control cannot generate the magnitude of χ later required. Returning to Fran De Aquino's actual nonlinear momentum-correlation equation repairs that contradiction. The corrected relation permits $\chi = 0$, $\chi < 0$, and unbounded negative magnitude. Consequently, a pair of negative-χ bodies can produce $G_{\text{eff}}/G = \chi_1\chi_2 \gg 1$ within the phenomenological force law, preserving the intended strong-coupling mechanism without inventing a positive branch that the correlation does not possess.

Three adversarial tests were applied. The algebraic test passes after replacing the erroneous reduction. The conservation test passes only conditionally after introducing an explicit measurable anomalous response coefficient rather than pretending ordinary low-power photon momentum suffices. The equivalence-principle test passes only if the effect is sharply state-dependent and vanishes in ordinary matter. These conditions make the theory falsifiable and complementary to De Aquino's work, but they do not constitute experimental verification.

References

1. F. De Aquino, "Correlation Between Gravitational and Inertial Mass: Theory and Experimental Test," arXiv:physics/0205089 (2002).
2. F. De Aquino, "Mathematical Foundations of the Relativistic Theory of Quantum Gravity," arXiv:physics/0212033 (2002; later revisions circulated).
3. F. De Aquino, "The Correlation Between Gravitation and Electromagnetism, Inertia and Unification," arXiv:physics/9905003 (1999).
4. EquationFarm, "Solving the Weak Gravitational Coupling Problem: A Comprehensive Analysis Based on Fran De Aquino's Relativistic Theory of Quantum Gravity and Gravity Control," April 19, 2026.
5. P. Touboul et al. (MICROSCOPE Collaboration), "MICROSCOPE Mission: Final Results of the Test of the Equivalence Principle," Phys. Rev. Lett. 129, 121102 (2022).
6. J. G. Lee et al., "New Test of the Gravitational $1/r^2$ Law at Separations Down to 52 μm," Phys. Rev. Lett. 124, 101101 (2020).

A Exact behavior of the De Aquino correlation

Starting from

$$\chi_{\mathrm{DA}} = 1 - 2(\sqrt{1+q^2} - 1), \tag{33}$$

expand for small q:

$$\sqrt{1+q^2} = 1 + \frac{q^2}{2} - \frac{q^4}{8} + O(q^6), \tag{34}$$

so

$$\chi_{\mathrm{DA}} = 1 - q^2 + \frac{q^4}{4} + O(q^6). \tag{35}$$

Thus the first deviation is quadratic rather than linear in normalized momentum transfer. For large $|q|$,

$$\chi_{\mathrm{DA}} = 3 - 2|q| - \frac{1}{|q|} + O(|q|^{-3}). \tag{36}$$

B Comparison with the discarded rational function

For the Grok paper's function

$$\chi_R(x) = \frac{1-x}{1+x}, \qquad x \geq 0, \tag{37}$$

we have

$$\frac{d\chi_R}{dx} = -\frac{2}{(1+x)^2}, \qquad \lim_{x\to\infty} \chi_R = -1. \tag{38}$$

Hence it is mathematically impossible for this function to support $\chi = -10$, -100, or -1000. Any paper using it and later asserting such values must be corrected.

C A compact pre-registration rule

Before an experimental run, record: the exact active-state definition S, predicted direction and harmonic of the gravitational signal, expected sign of χ, analysis bandwidth, environmental veto thresholds, calibration procedure, and stopping rule. A positive result is credible only if the predeclared model predicts data that the control-state and nuisance models do not.